Experiments in Control of Flexible Structures with Noncolocated Sensors and Actuators

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Experimental apparatus has been developed for physically testing control systems for pointing flexible structures, such as limber spacecraft, in the event that control actuators cannot be colocated with the sensors. (An example is the Galileo spacecraft, whose television camera at one end of a flexible beam must be pointed by torquing at the other end of the beam). With colocation, good stable control is very easy to achieve. With noncolocation it is extremely difficult, particularly if structural damping is very low and spacecraft stiffness and inertia values are uncertain and changing, as they are typically. For the apparatus we have built, structural damping ratios are less than 0.003, each basic configuration of sensor/actuator noncolocation is available, and inertias can be halved or doubled abruptly during control maneuvers, thereby imposing in particular a sudden reversal in the plant's pole-zero sequence, a most difficult problem for the controller. First experimental results are presented, including stable control with both colocation and noncolocation. The inherent robustness of the former is clearly seen, as is the great difficulty of achieving robustness for the latter. (Schemes for doing so are now being explored, and future experiments will establish what the best achievable robust but nonadaptive control is, and will develop adaptive control.) What we hope to contribute here is a "red flag" warning about noncolocated control of flexible structures: there are configurations, indeed, simple ones, for which there may be no practical alternative to adaptive control.

Introduction

A CRUCIAL problem for some flexible spacecraft is that the location of points at their extremities must be controlled, sometimes to very high precision, by torquing at some other point (e.g., the spacecraft center) that is separated from the first by sections of flexible structure. We shall call this "noncolocated control" for short. (The sensors and actuators used for control are not colocated, but are separated by flexible structures.) This is an extremely difficult control problem, for the opportunities for instability in such closed-loop systems are many and fundamental.

A particular case in point, and the initial stimulation for our research, is the Galileo spacecraft, which will be sent in the mid-1980's to study Jupiter and its moons, discovered by Galileo in the mid-1620's. The spacecraft spins slowly, but there is a beam-like structural system which is to be maintained inertially fixed by a motor at the hub. At the beam end is a television camera and other instruments which must be pointed, accurately and steadily on command, by a torque applied back at the hub and transmitted through the system of flexible beams to the televisin platform.

More generically, there will be large flexible spacecraft of many kinds on which the positions of many points at the spacecraft extremeties will need to be closely controlled, where the point positions can indeed be accurately measured (by optical means, for example), but where it is impossible or prohibitively costly to have an actuator at each point. Much greater design freedom is available if the technology exists for achieving precise, stable control by applying control torques at a distance, through the flexible structure.

This difficult control problem is further compounded by the facts that the physical damping in such large structures in space is apt to be very low, that the physical parameters are likely to be uncertain (prelaunch measurements at 1 g being difficult and inaccurate), and that the parameters will vary, sometimes by large amounts, as the spacecraft configuration is changed in the course of the mission.

It was discovered as early as 1965 (Refs. 1-3) that in the simpler case of colocated sensors and actuators ("colocated control") one can guarantee stability with relatively simple control laws. Because of this property, presumably nearly all of the theory to date for controlling flexible structures has begun by assuming colocation; and a considerable body of theory has, indeed, been developed for this case.[3-8] Reference 4 does contain a design method that is also applicable for noncolocated sensors and actuators using output feedback, but the method does not address the real question of stability in the presence of parameter variations (for which, in fact, stability is *not* guaranteed). Reference 9 also presents a design for control of a flexible beam with noncolocated sensors and actuators. The effects of control and observation spillover were taken into account in the report, but, again, the serious question of performance when parameters vary was not considered. In short, the references cited above do not address the question of robust control of flexible systems using noncolocated sensors and actuators, and none presents experimental data. Reference 13 does deal directly with noncolocated control of a flexible beam and presents experimental results. That work is complementary to the experiments with a multiple disk system presented here. Reference 10 directly addresses the Galileo problem of noncolocated control described above and presents a form of adaptive control for solving it, including both theory and simulation.

The details of why colocation leads to simple control and why noncolocation does not will be described presently. The object of our research is to develop and demonstrate some of the control understanding required to solve this problem—to effect precise, stable control in the presence of large changes in parameters for the general case of *noncolocated* sensors and actuators—using extensions of control theory and laboratory experiments to do so.

For our first experiments we have concentrated on laboratory structures that would have very low inherent damping. This is, of course, particularly difficult to achieve

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with beams in a 1-g field, and with air present, so we have developed the special apparatus we report on here, a torsional system with which we have been able to realize modal damping ratios of $\zeta \leq 0.003$. (We are also continuing our studies and experiments on the control of flexible beams.) The torsional system leads one, in a most direct way, to several fundamental insights, as we shall discuss presently.

We first describe our experimental system.

Experimental Apparatus

The laboratory system constructed for this investigation into control of noncolocated systems is shown pictorially in Fig. 1.

The system's plant consists of four steel disks, nine inches in diameter, with a thickness of one-half inch, attached firmly to a central, connecting torsion rod one-eighth inch in diameter. The system is suspended from the ceiling via a long length of piano wire to thrust relieve the bearings. Each disk is instrumented with an angular position sensor. (One is shown.) A brushless dc torque motor is installed at the third disk station, and provision exists for a second motor to be mounted on the lowest disk. A digital minicomputer is used to implement the control algorithms developed for the four-disk system and to collect the experimental data. A general block diagram of the control system is shown in Fig. 2.

Since the central experimental focus is to demonstrate control of noncolocated systems with uncertain parameters, the top disk is constructed so that its inertia can be varied while the system is under closed-loop control. This was achieved as shown in Fig. 3. The top disk consists of two pieces, an outer ring, and an inner disk; and lifting mechanism can be used to separate the ring from the disk in a fraction of a second. (The slight conical taper on the disk periphery and ring inner surface ensure that very little force is needed to separate the two, yet the large contact area prevents slipping when the ring and disk are engaged.) The major, fundamental effect of this parameter change upon the open-loop transfer function from the actuator to the noncolocated sensor on the bottom disk will be examined in the next section, Plant Dynamics.

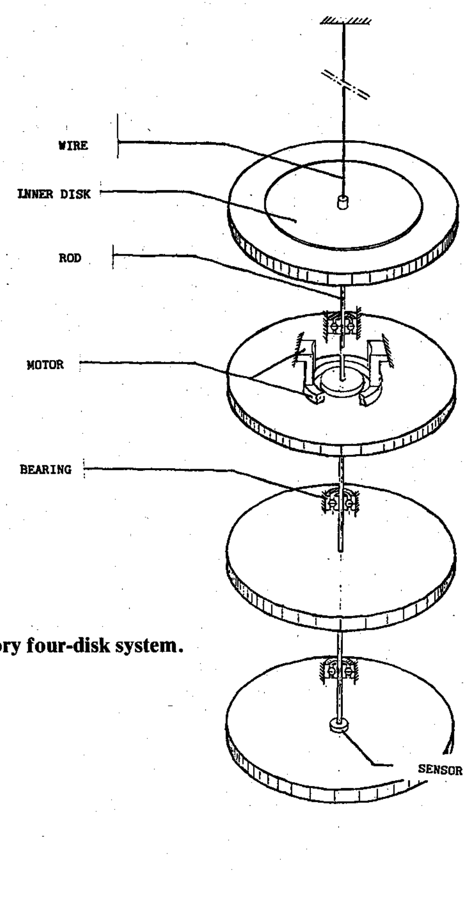

Fig. 1 Laboratory four-disk system.

The experimental design was governed by the requirement to incorporate: 1) sensor-actuator noncolocation, 2) a high degree of structural flexibility, 3) low inherent damping, and 4) the capability noted above to make a large, sudden change in a key parameter. At the same time, we wished to construct a system whose dynamics are well understood in order to insure unambiguous physical understanding of the closed-loop behavior.

Of course, a large space structure differs significantly from our laboratory model. The presence of an infinite number of modes and high-frequency modal uncertainty represent added complexity. However, the apparatus of Fig. 1 permits early useful experimentation upon a simple physical system that retains the basic ingredients of sensor/actuator noncolocation and large parameter changes, and achieves much lower damping than is possible with a laboratory beam.

Experimental results reported in this paper will show straightforward closed-loop control of the four-disk system in both a colocated configuration, Fig. 4a, and a noncolocated arrangement, Fig. 4b. The inherent inadequacies of the latter control will be demonstrated.

We begin by establishing the dynamics of the plant itself.

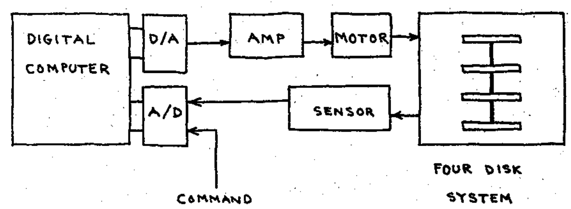

Fig. 2 Laboratory system block diagram.

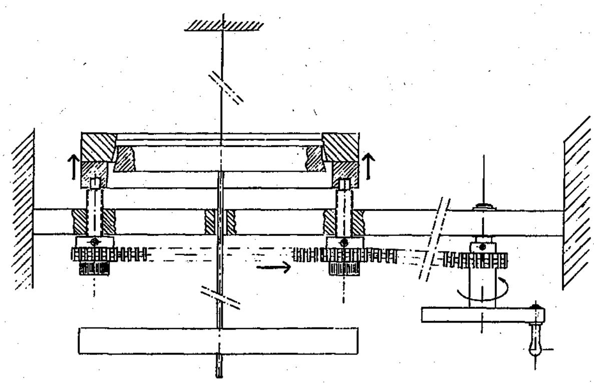

Fig. 3 Mechanism to change disk inertia abruptly.

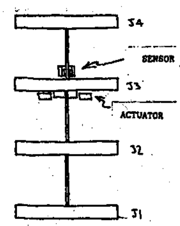

Fig. 4a Colocated sensor actuator arrangements.

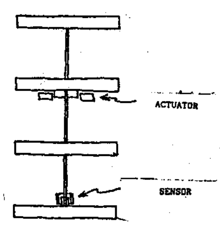

Fig. 4b Noncolocated sensor actuator arrangements.

Plant Dynamics

The top half of Fig. 5 shows the plant (open-loop) experimental response to an initial condition containing primarily the first vibration mode and the "rigid body" mode. The damping ratio of the first vibration mode is seen from this section of Fig. 5 to be about $\zeta_1 = 0.003$. (The frequency is seen to be 1.56 Hz.) A similar test, presented in the bottom half of Fig. 5, shows the second vibration mode damping ratio to be about $\zeta_2 = 0.002$. (The second mode frequency is seen to be about 2.9 Hz. The 10-s-period oscillation in Fig. 5 occurs, rather than the infinite period associated with a true rigid body mode, because of the wire used to suspend the system.)

Figure 6a shows the transfer functions for the two colocated input-output arrangements (actuator in to sensed displacement out) that are available on the four-disk system. The pole-zero diagram for each system is displayed at the right side of the figure. A most significant feature of all such colocated systems is the pole-zero alternation in the transfer function as one moves up the imaginary axis. This is the very desirable property that control designers have utilized so effectively for many years.

Figure 6b indicates the noncolocated transfer function from an actuator located at one end to a sensor at the opposite end. This arrangement leads to a transfer function with no zeroes.

Next, with the actuator moved to a disk one removed from one end, but still sensing the position of the disk at the opposite end, it is shown in Fig. 6c that a transfer function with a single zero between the first and second vibration poles is given. A different value for a plant parameter, such as inertia or stiffness, could yield instead a plant transfer function with a zero between the second and third vibration poles. It is this fundamental reversal in pole zero sequence (which we call "zero flipping") that makes stable control so particularly difficult to achieve in noncolocated control.

System identification tests were performed on the four-disk system to confirm the capability for the system to demonstrate a pole-zero flip. Two different techniques were found useful for this, sine wave tests and white noise tests using adaptive lattice filters. The sine wave tests are useful for directly identifying poles and zeroes of the four-disk system. This test was performed using a sine wave generator, frequency counter, and laboratory oscilloscope. For these tests, the sensor-actuator configuration of Fig. 4b was used. The system was driven at a frequency near a structural mode and a Lissajou figure displayed on the oscilloscope, using the sensor output on one channel and the plant input signal on the second channel. The vibration mode frequency could be determined very accurately in this way. This is true because the vibration modes are very lightly damped, so that at a resonance the Lissajou ellipse will have a 90-deg phase shift between input and output. This happens because the plant output will be maximized. The driving frequency could then be "dialed in" to find the vibration mode frequencies. In practice, it is very easy to align the ellipse by varying the frequency until a stationary, vertical image is obtained. The transfer function zeroes can be found in this way by varying the driving frequency to nullify the plant output.

The second test procedure involved the use of a laboratory test instrument, a Genrad 2515 Structural Analyzer. This device is actually a minicomputer equipped with real-time data acquisition hardware and software. This kind of instrument provides something of a "black box" approach to system identification, and will likely come into more and more prominence for structural testing via computer aided methods, due to the high utility of such methods. This test does not measure transfer function poles and zeroes directly. Rather, a data batch is gathered and then processed via a filtering algorithm to fit a linear transfer function to the data.

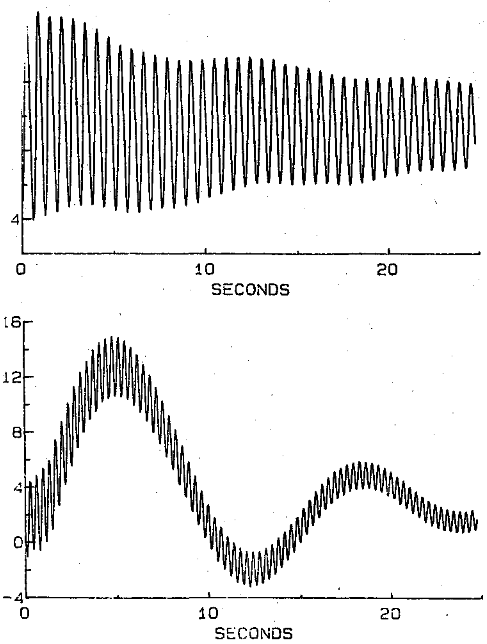

Fig. 5 Natural motion of plant above.

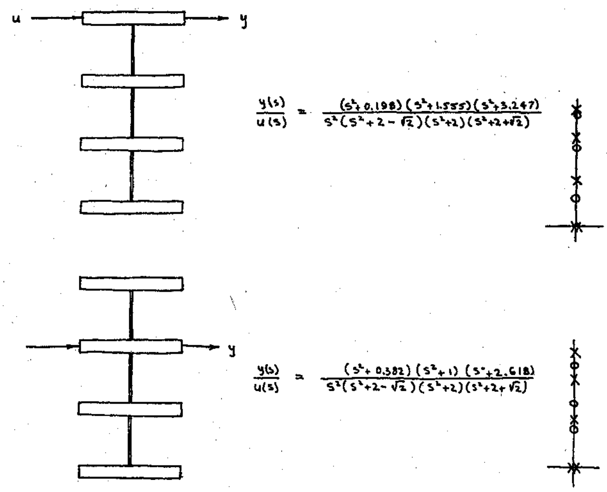

Fig. 6a Transfer functions for four-disk system: transfer functions of colocated sensor-actuator pairs.

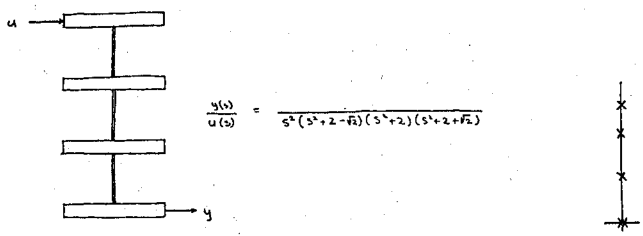

Fig. 6b Transfer functions for four-disk system: end-to-end transfer function.

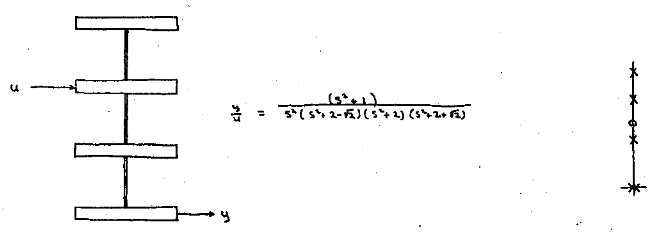

Fig. 6c Transfer functions for four-disk system: transfer function from inner actuator to end sensor.

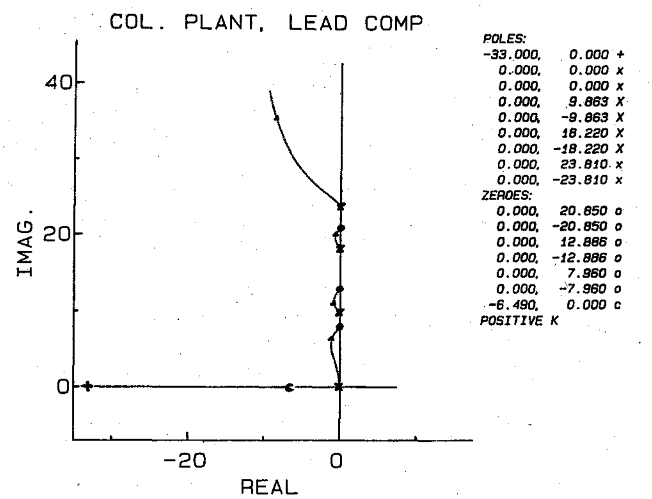

Fig. 7 Root locus for colocated control using lead compensation.

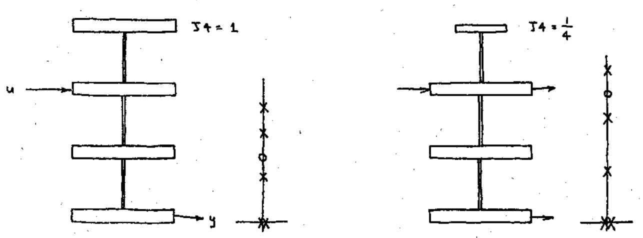

Fig. 8 Pole-zero flip with parameter change in noncolocated system.

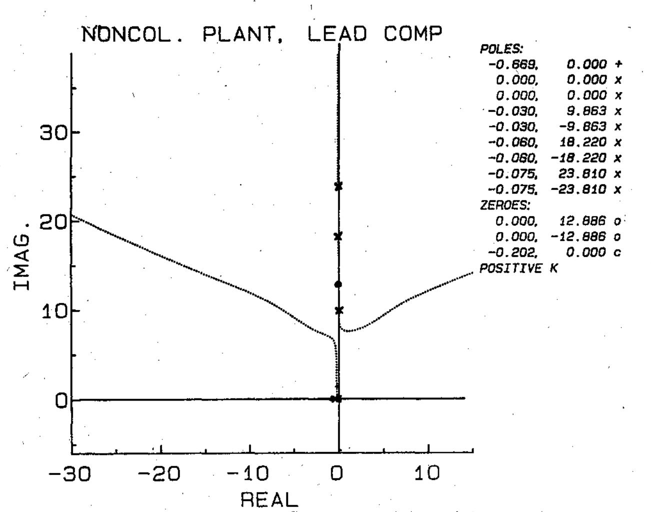

Fig. 9 Root locus for noncolocated control using lead compensation.

Test Results

Using the previous two test techniques the following poles and zeroes were identified.

For $J_4 = 1.0$, the sine wave test indicated $\omega_1 = 9.86$ (rad/s), $\omega_2 = 18.22$, and $\omega_3 = 23.81$. The zero was measured to be $z_1 = 12.89$. The Genrad equipment gave the following: $\omega_1 = 9.9$, $\omega_2 = 18.0$, and $\omega_3 = 24.0$. The zero was found to be $z_1 = 13.0$. The two tests differ by less than 1.0%. The ratios of these values match precisely the theoretical ratios of Fig. 6c.

For $J_4 = 0.25$, the sine wave test indicated $\omega_1 = 11.94$, $\omega_2 = 21.48$, and $\omega_3 = 29.34$. The zero was measured to be $z_1 = 25.9$. The Genrad test measured $\omega_1 = 12.0$, $\omega_2 = 22.0$, and $\omega_3 = 30.0$. The zero was found to be $z_1 = 24.0$. In this case, the difference between the tests is less than 2% for the vibration frequencies, and 4% for the zero frequency. Both tests confirm the pole-zero flip.

Control Design

To provide a clear context for the section on experimental results, we next discuss control design techniques for the four-disk system (using the root-locus format for exposition). Our approach has, of course, much in common with that required for controlling flexible beams. We first discuss control for the easier case that sensor and actuator are colocated.

Colocated System

The transfer function from the colocated actuator and sensor on the third disk is

$$\frac{y(s)}{u(s)} = 152\frac{(s^2 \pm 20.85j)\,(s^2 \pm 12.886j)\,(s^2 \pm 7.96j)}{(s^2)\,(s^2 \pm 23.81j)\,(s^2 \pm 18.22j)\,(s^2 \pm 9.863)} \qquad (1)$$

A simple lead network can readily be made to stabilize this system, as shown in the root-locus plot of Fig. 7. The lead network chosen has the transfer function

$$\frac{u(s)}{y(s)} = -8.3\frac{(s+6.50)}{(s+33.0)} \qquad (2)$$

The root-locus plot is drawn vs overall loop gain. The closed-loop system roots (indicated on the root locus) show that the rigid body mode is damped by 14%, the first vibration mode is damped by 12%, the second mode is damped by 2%, and the third mode is damped by 26%. The closed-loop bandwidth is 1 Hz (6 rad/s), or two-thirds of the first vibration mode, a reasonably fast system.

What is even more important is the inherent robustness of this control system. This is suggested from the root-locus, for the root-locus lies entirely in the left half, or stable region of the s plane. The lead network is effective for providing fast, robust control for the colocated system because the colocated transfer function (from torque to motion at the sensor) always has alternating poles and zeroes (Fig. 6). Thus, it is easy to phase stabilize every vibration mode without accurate knowledge of the vibration-mode frequencies or damping ratios.

This result is clear on physical grounds: we know that the plant can be stabilized by adding passive dashpots. A dashpot removes energy from the system by providing a force proportional to rate, and the force is applied at the same point where the rate occurs. Using pure rate feedback would perform exactly the same function, cancelling one of the rigid body poles and yielding a root-locus in which the departure angle at every pole would be 180 deg. The actual lead network is part of an active controller that causes the closed-loop system to behave nearly as a passive system, providing colocated rate feedback plus position feedback to control the rigid body mode. [The pole in Eq. (2) makes the control realizable by providing high-frequency rolloff.]

The simple lead network thus yields stable compensation for large parameter variations, and with a relatively high closed-loop bandwidth.

Noncolocated System

Turning now to the noncolocated case, we find a much different situation. To understand it, we will compare the results of employing two kinds of compensator: first a lead network (as for the colocated case above), and then a full order optimal compensator.

The nominal plant transfer function is

$$\frac{y(s)}{u(s)} = 827{,}000\,\frac{(s^2 \pm 12.886j)}{(s^2)\,(s^2 \pm 23.81j)\,(s^2 \pm 18.22j)\,(s^2 \pm 9.863)} \qquad (3)$$

The salient new feature with which the controller must now cope is the striking effect that a parameter variation can

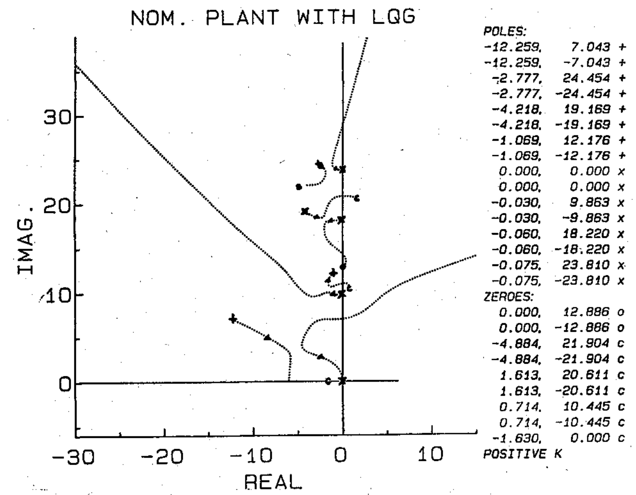

Fig. 10 Root-locus for noncolocated control using eighth order optimal compensation: nominal plant.

produce in the plant transfer function of a noncolocated system. The effect is illustrated in Fig. 8: as the value of inertia J_4 is decreased, a pole-zero "flip" can occur! That is, the transfer function zero that begins between the first and second vibration poles in the system with $J_4 =$ unity moves to a new location between the second and third vibration poles when J_4 is reduced to 0.38% of its nominal value or less. Since in general the nominal value of J_4 could actually be near 0.38 to begin, a very small percentage change can in fact cause a pole-zero flip. (For the experimental results reported here, J_4 was varied between its nominal value, one-half its nominal value, and one-fourth its nominal value. As discussed in a previous section, a pole-zero flip occurs in the four-disk system when J_4 is varied between one-half and one-fourth the nominal value.) This poses a most difficult control problem, implying the need for a compensator that can provide over 180 deg of phase margin near the frequency range where the pole-zero flip occurs. Thus, a controller designed for one value of J_4 will surely go unstable if J_4 subsequently takes on the other value, unless the possibility of the pole-zero flip is accounted for meticulously in the control design. And that is not easy.

"Pole-zero flipping" can be serious. It is true, of course, that if a small parameter change can cause a pole-zero pair to flip, this implies the pole and zero were close, or nearly cancelling in the open-loop system from control torque to sensor motion, so that this mode, therefore, does not contribute heavily to the system response to a command. Commands, however, are not the only system input. There will also be disturbances acting on the structure. Moreover, because inherent damping is so low in large space structures, the pole need not be shifted very far to become unstable. To reduce the plant model by truncating this mode may well be to ask for disaster, for in the presence of even mild parameter uncertainty, the unstable mode may be discovered for the first time after the spacecraft has been launched. Instead, the control system should be designed to have a stabilizing effect, or at most no effect, on every pole, even if it is possible to improve the pole location only a small amount because of the nearby zero.

Lead Compensation

Using lead compensation, we can, in fact, design a very-low-bandwidth controller for the noncolocated system. The control design basically treats the plant like a rigid body. The crossover frequency is chosen so that even the first vibration mode is gain stabilized. To achieve a stable system, we must rely entirely on inherent damping in this case, because the lead network always destabilizes at least one vibration mode, Fig. 9. Further, we must have a good idea of the vibration mode frequencies and damping ratios even when designing a low bandwidth controller.

The control design is essentially a tradeoff to see how far the rigid body roots can be moved before the flexible modes are destabilized; the result inevitably will be slow control. This situation is thus markedly different from the colocated case, even for a low-performance design.

Specifically, the lead network used in Fig. 9 has the transfer function

$$\frac{u(s)}{y(s)} = -0.389\frac{(s+0.2024)}{(s+0.6692)} \tag{4}$$

In the resulting closed-loop system, the modified rigid body poles are at $s = -0.29, \pm 1.75j$, only about 17% of the first plant mode frequency, compared with 66% for the colocated case (Fig. 9).

To achieve any higher performance in the noncolocated case, a higher order compensator must be used. Then faster response can be achieved, but robustness will be quite unacceptable, as we shall see.

Linear Quadratic Gaussian Design

An optimal control design, using linear quadratic Gaussian (LQG) synthesis techniques can yield a higher performance system *if the plant parameters are precisely known*. The compensator will be eighth order, the same order as the plant to be controlled. The root locus of Fig. 10 shows that the compensation consists of lead at low frequency and notch filters at each structural mode. (Reference 12 provides a complete discussion about compensator transfer functions in optimally controlled systems.)

It is the use of structural notches that allows the closed-loop system to achieve higher bandwidth than the simple lead network. Each notch consists of a pole-zero pair. The compensation basically cancels the structural pole with the notch zero and substitutes a more heavily damped compensator pole in its place. This allows the compensator to have higher gain and thus move the rigid body poles further. On the root-locus plot the effect of the notch filters is characterized by the fact that each plant structural pole moves to the left, or towards the stable region of the s plane, while the notch poles move to the right. Again, the root locus was drawn by varying the compensator gain. The compensator transfer function in the example is

$$\begin{aligned}\frac{u(s)}{(y)s} = -124\{\,[(s+4.88\pm 21.9j)\,(s-1.613\pm 20.6j)\\ \times (s-0.714\pm 10.4j)\,(s+1.63)\,]/[s+12.259\pm 7.043j)\\ \times (s+2.78\pm 24.454j)\,(s+4.22\pm 19.17j)\\ \times (s+1.069\pm 12.176j)\,]\,\}\end{aligned} \tag{5}$$

The closed-loop poles are: $s = -2.0 \pm 23.9j$, $-2.2 \pm 18.5j$, $-1.5 \pm 9.8j$, $-4.5 \pm$? 8*j* The closed-loop bandwidth is 45% of the first vibration mode.

However, to be effective, the notches must be tuned very precisely. The resulting controller is therefore sensitive to any change in a plant parameter, that is, to a change in either the plant's vibration frequency or its damping. That is, when the model is correct, the control system should meet its specifications *but only so long as the system's parameters stay close to their assumed values.* If they do not, the closed-loop system may well become unstable. Figure 11 is a plot of the locus of closed-loop roots when the compensator [Eq. (5)] designed for the nominal system (J_4) is applied to a system in which $J_4 = 0.25$. In this case, the pole-zero flip causes the closed-loop system to become unstable well before the nominal compenator gain is reached. In fact, the first vibration mode is destabilized imemdiately.

The nominal compensator can in fact stabilize the system *only when J_4 is within 10% of the design value,* and even then, closed-loop performance is severely lowered.

Good noncolocated control can be very hard to achieve.

Experimental Results

We now present results from some physical experiments which demonstrate, some perhaps for the first time, each of the above fundamental concepts about flexible spacecraft control, namely: 1) ease of achieving fast, robust control with colocation, 2) the great difficulty, with noncolocation, of achieving control that is merely stable, how slow such control must be, and how small a change in plant parameters can make it unstable.

Figures 12-14 show the behavior of the laboratory four-disk system under the several forms of closed-loop control described in the section on control design.

Colocated Control

Nominal Plant

Figure 12a shows the response to an initial condition of the system in which closed-loop control is achieved using a colocated sensor and actuator. The system was initially excited by simply rotating it off zero and shaking one of the disks by hand to excite the rigid body mode and the first vibration mode. The system is uncontrolled during the first 3 s of the figure, at which point the control is suddenly turned on. The commanded position is zero, so the figure shows the regulation capability of the controller. The system natural frequency is 1.2 Hz, or about 70% of the first vibration mode. This agrees quite well with the design given by the root locus of Fig. 7. The response dies out in just over three vibration cycles. There is a small amount of second and third mode contained in the output, as well as some evidence of output quantization.

Figure 12b shows the response to an initial condition of the system which contains primarily the second vibration mode.

Step Response

Figure 12c shows the response of the same system to a 10-deg step change in commanded position. The system natural frequency can again be seen to be about 70% of the first vibration mode, which is in agreement with the design given by the root locus of Fig. 7. The damping ratio is approximately 12%, also close to the predicted value of 14%.

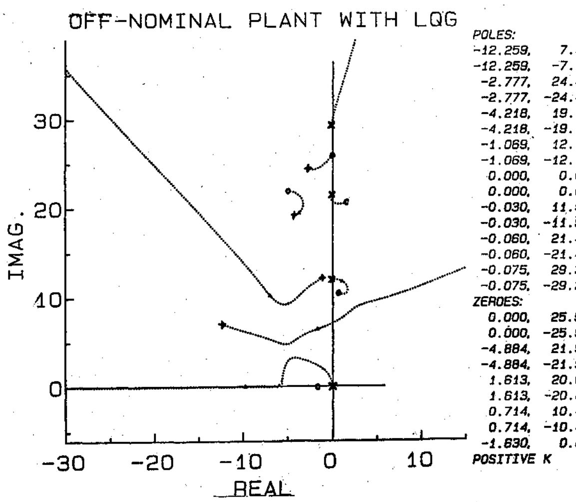

Fig. 11 Root locus for nominal optimal compensator applied to off-nominal plant.

Affect of Parameter Change

Figure 12d shows the response of the four-disk system in which J_4 is only one-fourth the nominal value, but using the lead compensation designed for the nominal system. The system response is essentially identical to the nominal case, thus demonstrating the robustness and high performance obtainable (Fig. 8) when the sensor and actuator of the control system are colocated.

Noncolocated Control

Figures 13 and 14 show the response of the four-disk system in which closed-loop control is achieved using noncolocated sensor and actuator.

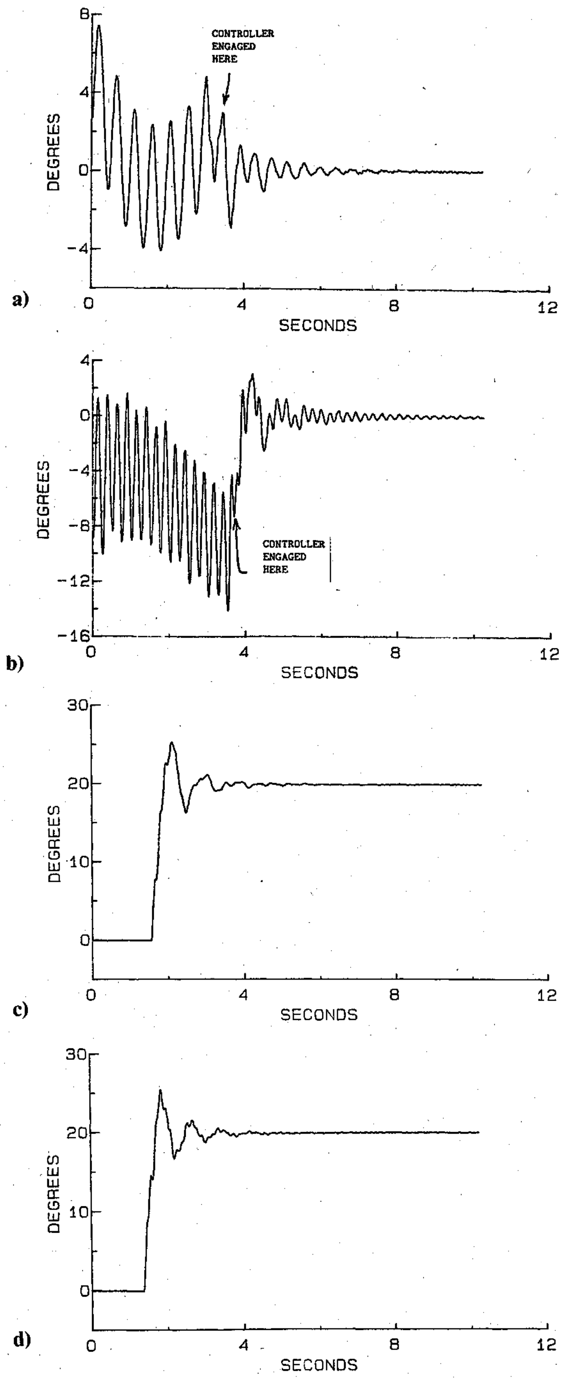

Fig. 12 Experimental response with colocated control using lead compensation: a) Nominal plant (response to first mode initial condition); b) Nominal plant (response to second mode initial condition); c) Nominal plant (response to step command); d) Off-nominal plant (response to step command).

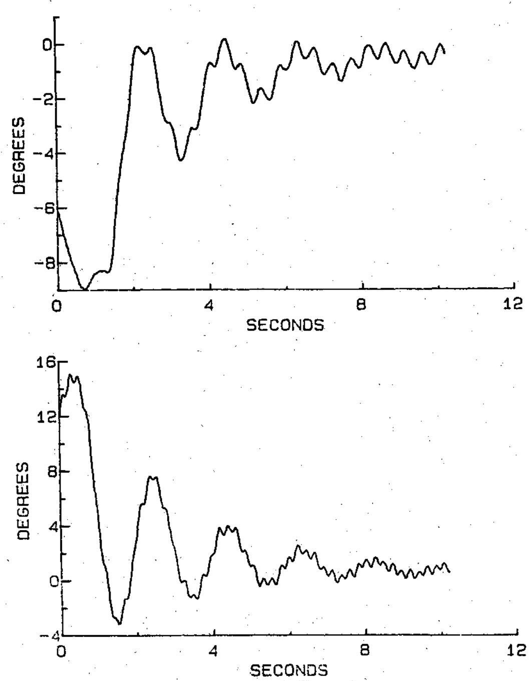

Fig. 13 Response of noncolocated system, showing slow response: a) Nominal system with lead compensation; b) Off-nominal system with nominal lead.

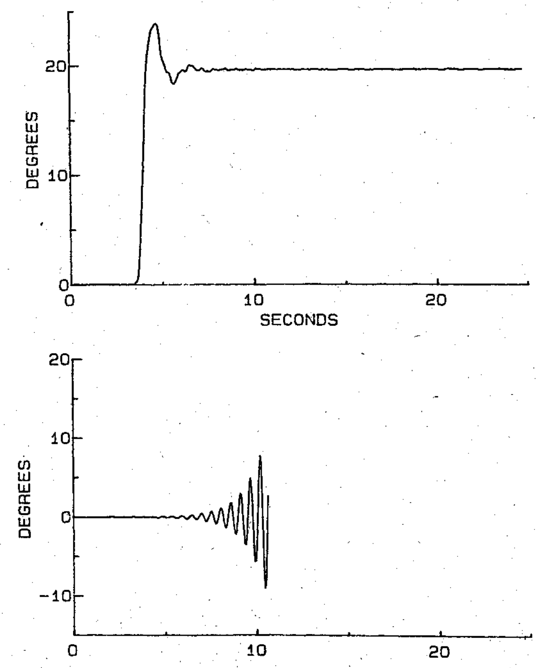

Fig. 14 Response of noncolocated system with eighth order LQG compensator: a) Nominal plant with nominal compensator; b) Off-nominal plant with nominal compensator showing instability.

Simple Lead Controller: Nominal Plant

Response with the simple lead compensation of Fig. 9 is shown in Fig. 13a. In this case, a bandwidth of about only 10% of the first vibration frequency is possible, even when the plant parameters have exactly their nominal values. This agrees with the prediction of Fig. 9. The response contains a component at the first vibration mode which does not die out perceptibly in 10 s. This indicates the predicted inability of the low bandwidth controller to damp the vibration modes. (See Fig. 5, the plant's free response.)

Effect of Parameter Change

Figure 13b shows the effect of a parameter change upon this system's stability. In this case, the system remains barely stable when J_4 is changed from its nominal to one-half its nominal value, even for the extremely slow system achieved in Fig. 13a.

Optimal Controller

Figure 14 shows experimental performance when the eighth-order LQG compensator of Fig. 10 is used. Figure 14a shows the closed-loop step response. The response has about 10% overshoot and a rise time of about 1 s. The steady-state performance has somewhat more "jitter" than did the colocated lead network, but is is much faster than the response obtainable with the noncolocated lead network. (The jitter could be reduced at the expense of slower response by adjusting the weighting factors.)

LQG Controller

The performance shown in Fig. 14a is available from the LQG compensator *only* when the plant parameters are precisely known. This is demonstrated dramatically in Fig. 14b, which shows the unstable response when the LQG compensator designed for the system with $J_4 = 1.0$ is applied to the system in which $J_4 = 0.25$. The controller was turned on with the system in its nominal configuration. After 2 s, the parameter J_4 was changed while the controller was on, and the subsequent rapidly growing unstable motion recorded. The frequency of the unstable vibration is 1.78 Hz. The predicted value (from the root locus of Fig. 10) is 1.8 Hz. The initial rate of growth is also very close to the predicted rate: 0.9 s observed vs 0.93 s doubling time predicted. The LQG system is not at all robust in this case, as is shown dramatically in Fig. 14b where, clearly, the closed-loop system becomes unstable when the value of J_4 is changed.

In a subsequent series of experiments we will focus directly on the stability vs robustness question. We will seek to establish the absolute best, i.e., most robust, nonadaptive control that is achievable, especially in the case where a pole-zero flip can occur. (These experiments will employ some new approaches to fixed compensator design.) With this result as a base, we will then begin to apply adaptive control techniques to achieve, finally, performance that is acceptable, even with a pole-zero flip.

Summary and Conclusions

This paper has described a new experimental apparatus for investigating control laws for large flexible spacecraft. The initial series of experiments has been intended to demonstrate the difficulties associated with active control of large space structures, particularly when the sensors and actuators are noncolocated. Such systems will have many low-frequency vibration modes and very low inherent damping. The control system will be designed using a model of the structure which contains uncertainty, and the actual plant parameters will vary with time, so that the control system needs to be robust.

The laboratory system was designed to provide a control-system test bed exhibiting each of the above characteristics. The system possesses three vibration modes plus a rigid body mode, and is instrumented to allow control configurations with either colocated or noncolocated sensors and actuators. A key system parameter can be changed while the system is under closed-loop control, so that robustness of the control design can be most severely tested. Natural damping of the system's vibration modes is less than 0.3%.

What we have shown in the initial experiments reported here is that, in the case where sensors and actuators are noncolocated, any controlled, flexible system may well be extremely sensitive to the actual values of system parameters, so that quite sophisticated techniques will be needed to achieve fast, stable, robust control. When the sensors and actuators are noncolocated, the control system must account for the presence of many vibration modes. Modal damping ratios and vibration frequencies must be known accurately or identified continually, because the controller will invariably destabilize some of the high-frequency modes, even when the plant is known, so that the typically low values of inherent damping will limit achievable performance greatly.

Finally, systems with sensor-actuator noncolocation can exhibit "pole'zero flipping" when parameters vary (while colocated systems always have alternating poles and zeroes even when parameters vary greatly). It is suggested that control system designers be most wary of these conditions.

The next series of experiments will apply parameter optimization tools to investigate the capability of the most advanced robust-control design techniques to cope with such difficult problems as large parameter changes and pole-zero flips. This work will provide a baseline to assess definitively the circumstances in which only adaptive control techniques can supply robust control for flexible spacecraft. One possibility for such adaptive control is suggested in Ref. 10; and Ref. 14 reports demonstrations of such adaptive control schemes applied to a simple torsional system. The laboratory four-disk system described in this paper will also be modified to create a system with nearly equal vibration frequencies. This case presents an extreme challenge for adaptive control methods which rely upon frequency identification methods, and thus is most relevant for future work in control of large space structures.

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